# Combustion Efficiency

## Gas Turbine Combustion Short Course

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- Reaction rate and evaporation rate-controlled systems

- Numerical example

# Combustion Efficiency

- Combustion efficiency:

$$\eta_C = \frac{WF(burned)}{WF(total)}$$

$$\eta_C = f(air\ flow\,rate)^{-1}\left(\frac{1}{evaporation\,rate} + \frac{1}{mixing\,rate} + \frac{1}{reaction\ rate}\right)^{-1}$$

- Combustion inefficiency:
  - Represents waste of fuel
  - Source of harmful/undesirable pollutants (CO and UHC)

- Typical values:
  - > 99% for all operating conditions
  - 75% - 80% for altitude relight
    (safety requirement to compensate for narrower stability limits)

# Reaction Rate-Controlled Systems:
## Burning Velocity Model

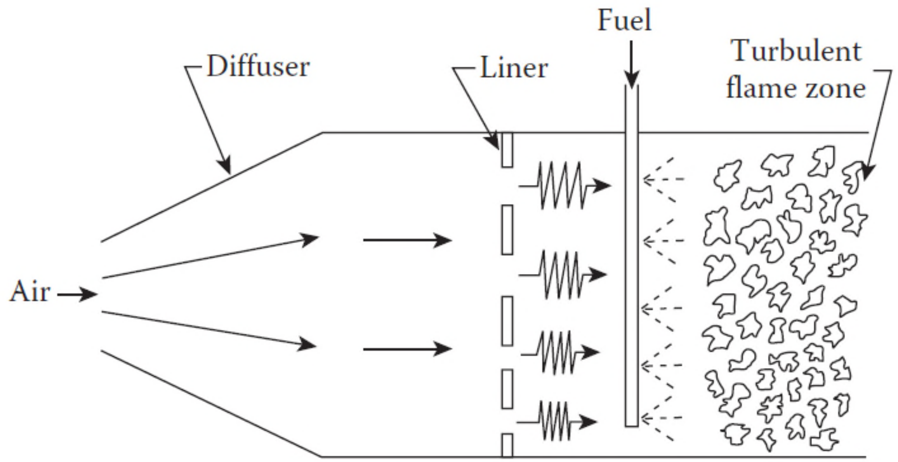



Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 3rd Edition, McGraw Hill

- Combustion zone similar in structure as turbulent flame brush of Bunsen burner
- Combustion efficiency is a function of the ratio of turbulent burning velocity to the velocity of the mixture entering the combustion zone
- Assumptions:
  - Evaporation rate and mixing rate assumed to be infinitely fast
  - All the fuel that burns does so completely
  - Inefficiency arises when some of the mixture passes through the combustion zone without being entrained by turbulent flame front

# Burning Velocity Model:

## Derivation of the $\theta$ Parameter

$$\eta_C = \frac{heat\ released\ in\ combustion}{heat\ available\ in\ fuel} = \frac{\rho_g A_f S_T Cp_g \Delta T}{q m_a LHV}$$

$$\begin{bmatrix} Cp_g \times \Delta T = q \times LHV \\ A_f \propto A_{ref} \\ m_a = \rho_g \times A_{ref} \times U_{ref} \end{bmatrix} \Rightarrow \quad \eta_C \propto \frac{S_T}{U_{ref}}$$

| | |
|---|---|
| $A_f$ | – Flame area |
| $A_{ref}$ | – Combustor reference area (area at maximum diameter) |
| $Cp_g$ | – Gas Specific heat at constant pressure |
| LHV | – Lower heating value of fuel |
| $m_a$ | – Inlet air mass flow rate |
| q | – Fuel to air ratio by mass |
| $S_T$ | – Turbulent flame speed |
| $U_{ref}$ | – Combustor reference velocity |
| $\Delta T$ | – Temperature rise due to combustion |
| $\rho_g$ | – Density of the gas |

# Burning Velocity Model:
## Derivation of the θ Parameter

$$\eta_C \propto \frac{S_T}{U_{ref}}$$

Expressing:

- $U_{ref}$ as a function of $m_a$, $P_3$ & $A_{ref}$
- $S_T$ as a function of laminar burning velocity and turbulence intensity (related to $\Delta P_L$)

$$\eta_C = f\left[\frac{P_3 A_{ref}\left(P_3 D_{ref}\right)^x \exp\left(T_3 / b\right)}{m_a}\right]\left[\frac{\Delta P_L}{Pt_{ref}}\right]^{0.5x}$$

b – Temperature dependence of reaction rates
$D_{ref}$ – Maximum diameter of combustion casing
$Pt_{ref}$ – reference total pressure
$T_3$ – Combustor inlet temperature
x – constant
$\Delta P_L$ – Liner pressure differential

# Burning Velocity Model:

## Derivation of the θ Parameter

$$\eta_C = f\left[\frac{P_3 A_{ref}\left(P_3 D_{ref}\right)^x \exp\left(T_3 / b\right)}{m_a}\right]\left[\frac{\Delta P_L}{Pt_{ref}}\right]^{0.5x}$$

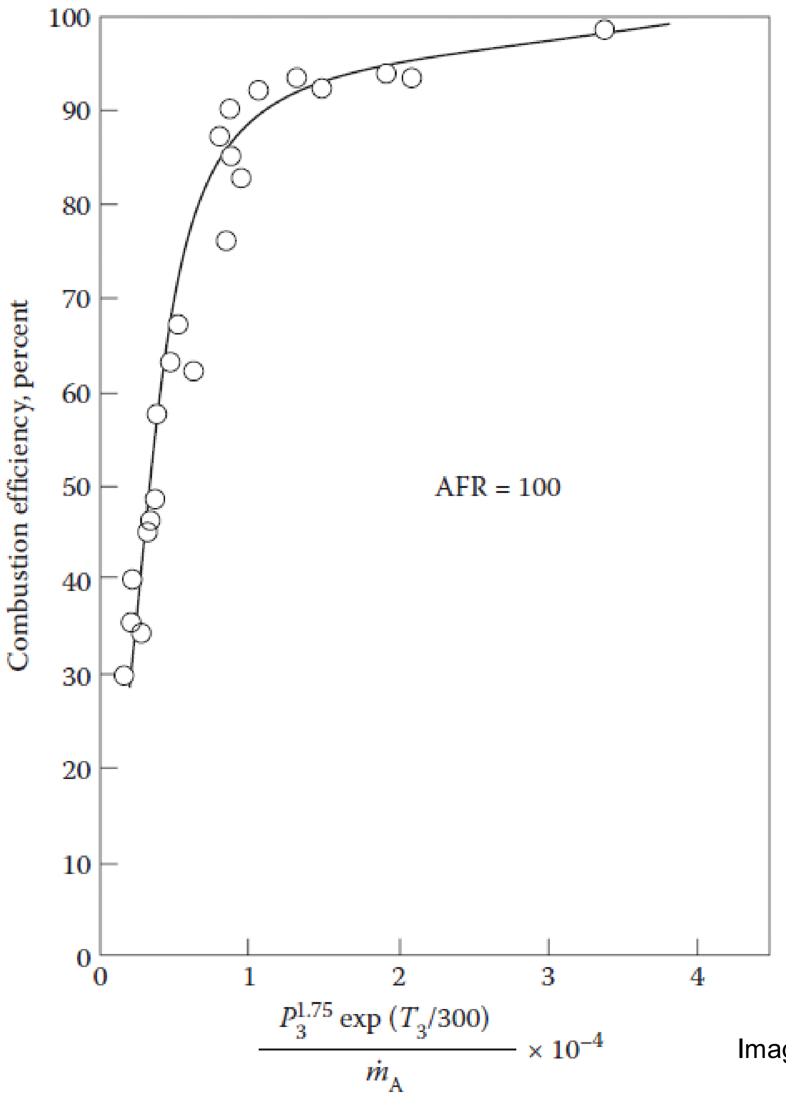
- Not possible to derive θ parameter expression analytically
- Values of b and x have been derived from experiments
  - Derived value of b ≈ 300
  - Derived value of x ≈ 0.75
- Experimental evidence suggests inclusion of $\Delta P_L$ is meager and does not vary much

$$\eta_C, \theta = f(\theta) = f\left[\frac{P_3^{1.75} A_{ref} D_{ref}^{0.75} \exp(T_3 / 300)}{m_a}\right]$$

Theta "θ" Parameter

# Burning Velocity Model:

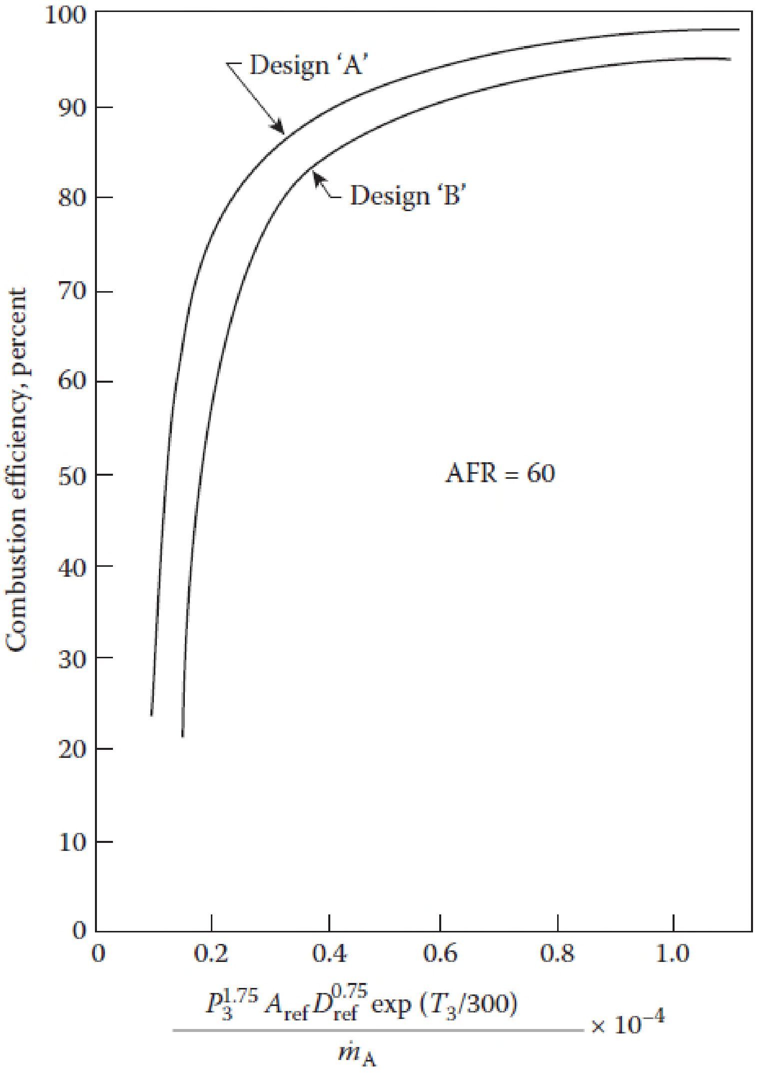
## The θ Parameter




- θ useful for reducing amount of rig testing required to evaluate new designs
- Few test points required to establish complete performance curve
- Possible to predict η values at flow conditions that lie outside the capacity of test facility
- Provides method of scaling combustor dimensions and operating conditions so any changes in performance can be attributed to design differences

Image courtesy of: Lefebvre. A. H., 2010, “Gas Turbine Combustion”, 3rd Edition, McGraw Hill

# Burning Velocity Model:
## The θ Parameter




- Design A is clearly superior:
  - For any value of $\eta$, $\theta$ is lower $\Rightarrow$
  - Under any operating conditions of $m_a$, $P_3$ & $T_3$ for $\eta_A = \eta_B$, Design A can be made smaller in size

Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 3rd Edition, McGraw Hill

# Mixing Rate-Controlled Systems

- If evaporation and reaction rates are assumed to be infinitely fast, then:

$$\eta_C = f\left(\frac{mixing\ rate}{air\ flow\ rate}\right)$$

$$mixing\ \ rate = (eddy\ \ diffusivity)\times(mixing\ \ area)\times(density\ \ gradient)$$

$$mixing\ \ rate \propto (lU_J)\times(l^2)\times\left(\frac{\rho}{l}\right)$$

$$mixing\ \ rate \propto \rho \times U_J \times l^2$$

l – Turbulent length scale
(measure of size of large energy-containing eddies in a turbulent flow)
$U_J$ – Turbulent velocity in air jet
ρ – Density

# Mixing Rate-Controlled Systems

$$mixing\ rate \propto \rho \times U_J \times l^2$$

Substituting for: $\Delta P_L \propto U_J{}^2 \times \rho$ and $\rho = \dfrac{P_3}{R \times T_3}$

$$\Rightarrow \quad mixing\ rate \propto \left(\frac{P_3 l^2}{T_3{}^{0.5}}\right) \times \left(\frac{\Delta P_L}{P_3}\right)^{0.5}$$

$$\Rightarrow \quad \eta_m = f\left(\frac{P_3 A_{ref}}{m_a T_3{}^{0.5}}\right) \times \left(\frac{\Delta P_L}{P_3}\right)^{0.5}$$

(Assuming turbulence scale is proportional to combustor size)

# Evaporation Rate-Controlled Systems:
## Mass flow Rate of Evaporated Fuel

- When mixing and reaction rates are fast enough ⇒ evaporation may be the rate controlling step

$$m_f = 1.33\,\pi\,n\,D\left(k / C_p\right)_g \ln(1+B)\left(1+0.25\,\mathrm{Re}_D^{\;0.5}\right)$$

B – Mass transfer number (or Driving Force)
(determines rate of mass transfer across a medium)
$C_p$ – Specific heat at constant pressure
D – Sauter mean diameter (SMD)
(diameter of drop having the same volume/surface area ratio as the entire spray)
k – Thermal conductivity
(measure of the ability of a material to conduct heat)
$m_f$ – Mass flow rate of evaporated fuel
n – number of drops of fuel
$Re_D$ – Reynolds number of droplet (corresponds to fluctuating velocity)

$$\left(\mathrm{Re} = \frac{Body\ Forces\ (reflects\ velocity\ \&\ momentum\ effects)}{Viscous\ Forces\ (cause\ fractional\ pressure\ losses)}\right)$$

# Evaporation Rate-Controlled Systems:
## Mass flow Rate of Evaporated Fuel

$$m_f = 1.33\,\pi\,n\,D\left(k/C_p\right)_g \ln(1+B)\left(1+0.25\,\mathrm{Re}_D^{\,0.5}\right)$$

$$q_c = \frac{n(\pi/6)D^3\rho_f}{V_c\rho_g} \qquad\qquad n = \left(\frac{6}{\pi}\right)\left(\frac{\rho_g}{\rho_f}\right)\left(\frac{V_c}{D^3}\right)q_c$$

n – number of drops of fuel
$\rho_f$ – Fuel density
$\rho_g$ – Gas density
$q_c$ – FAR in combustion zone

By substitution:

$$m_f = 8\left(\rho_g/\rho_f\right)\left(k/C_p\right)_g\left(V_c/D^2\right)q_c \ln(1+B)\left(1+0.25\,\mathrm{Re}_D^{\,0.5}\right)$$

# Evaporation Rate-Controlled Efficiency:
## Combustion Efficiency

$$m_f = 8(\rho_G / \rho_F)(k / C_p)_g (V_c / D^2) q_c \ln(1+B)(1 + 0.25 \mathrm{Re}_D{}^{0.5})$$

$$\eta_{ce} = \frac{m_f t_{res}}{\rho_G V_c q_c}$$ (ratio of mass of fuel evaporated to mass of fuel supplied)

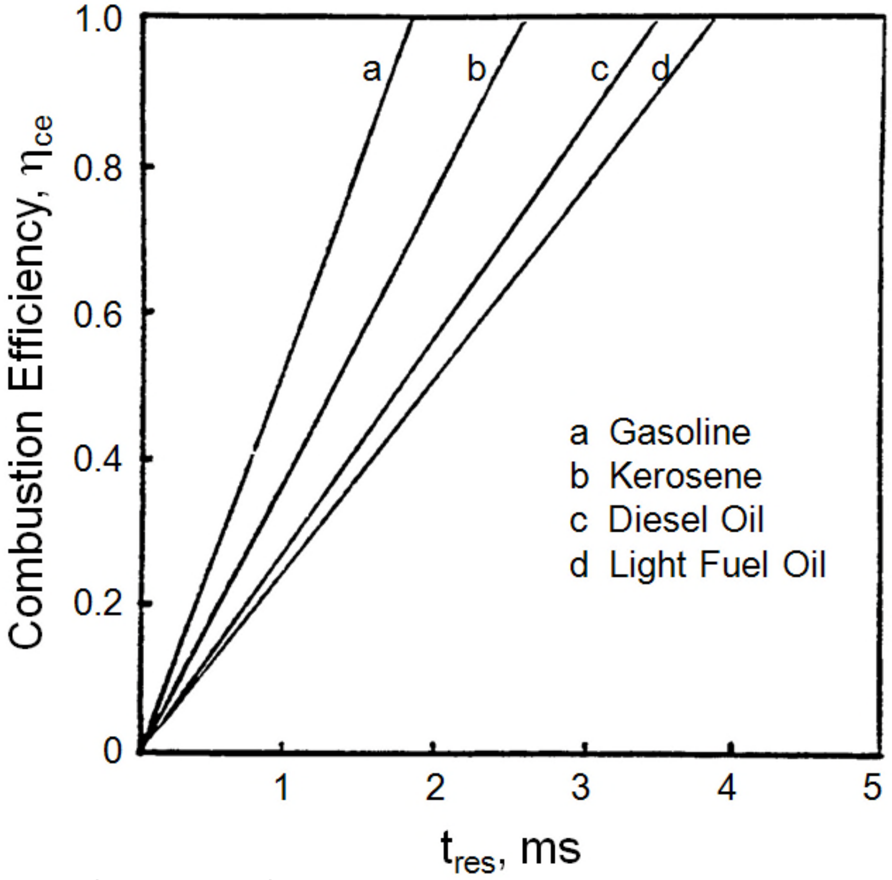
$t_{res}$ – residence time

- NB: For sufficiently large $t_{res}$ it is possible for $\eta_{ce}$ to exceed unity
  ⇒ Evaporation is not limiting to combustion efficiency and $\eta_{ce} = 1$

By substitution:

$$\eta_{ce} = \frac{8 (k / C_p)_g \ln(1+B)(1 + 0.25 \mathrm{Re}_D{}^{0.5}) t_{res}}{\rho_f D^2}$$

# Evaporation Rate-Controlled Efficiency:
## Influence of Fuel Type and Residence Time




Influence of residence time on evaporation rate-controlled combustion efficiency
($T_g$ = 2300K, D = 60μm)

Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 2nd Edition, McGraw Hill

| Fuel | Density (kg/m$^3$) | Mass Transfer Number (B) |
|---|---|---|
| Gasoline (JP 4) | 692 | 6.10 |
| Kerosene (JET- A) | 775 | 3.75 |
| Diesel Oil (DF2) | 900 | 2.80 |
| Light Fuel Oil | 930 | 2.50 |
| Heavy Fuel Oil | 970 | 1.50 |

Fuel Properties Table

# Evaporation Rate-Controlled Efficiency:
## Influence of Drop Size and Residence Time

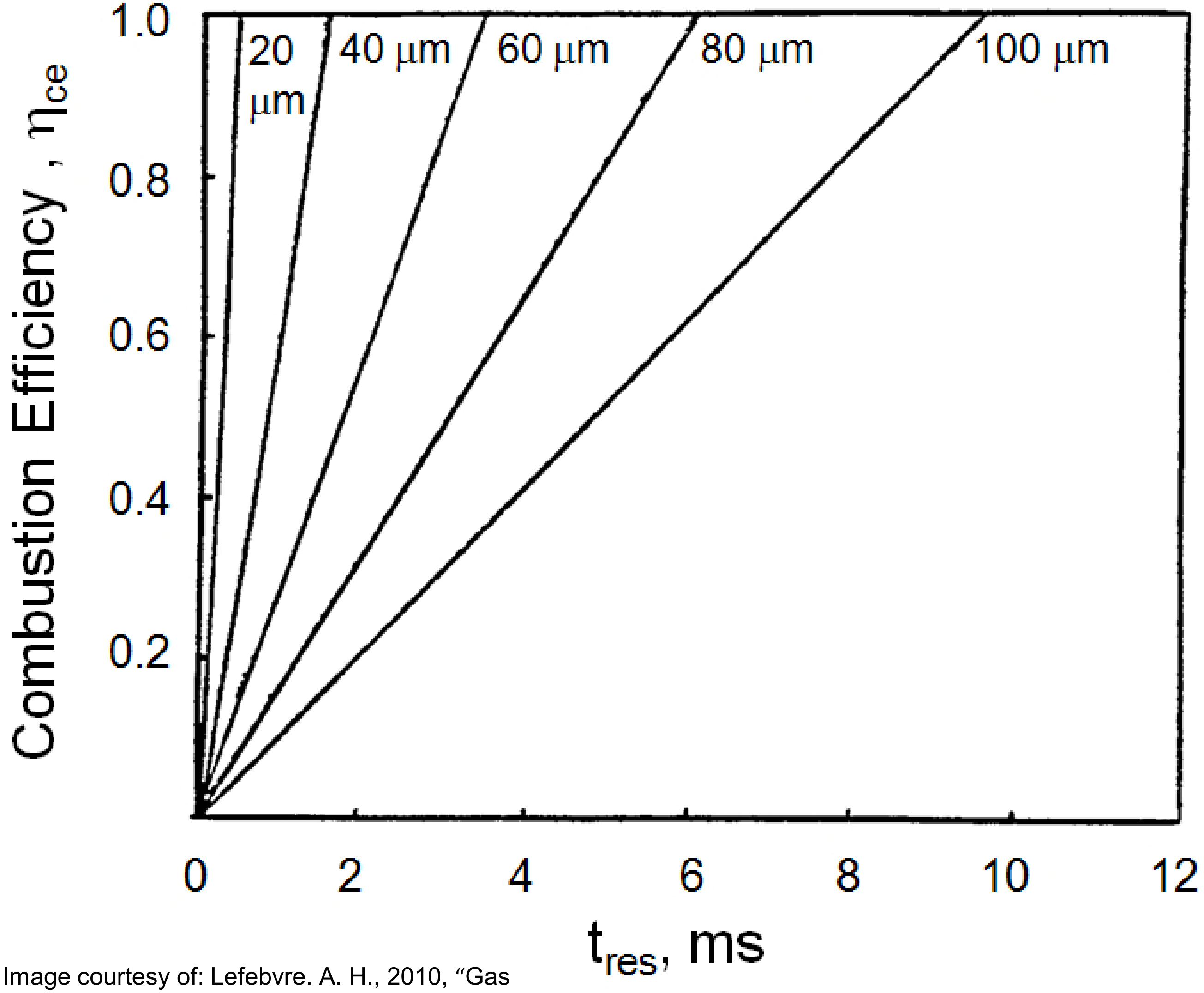
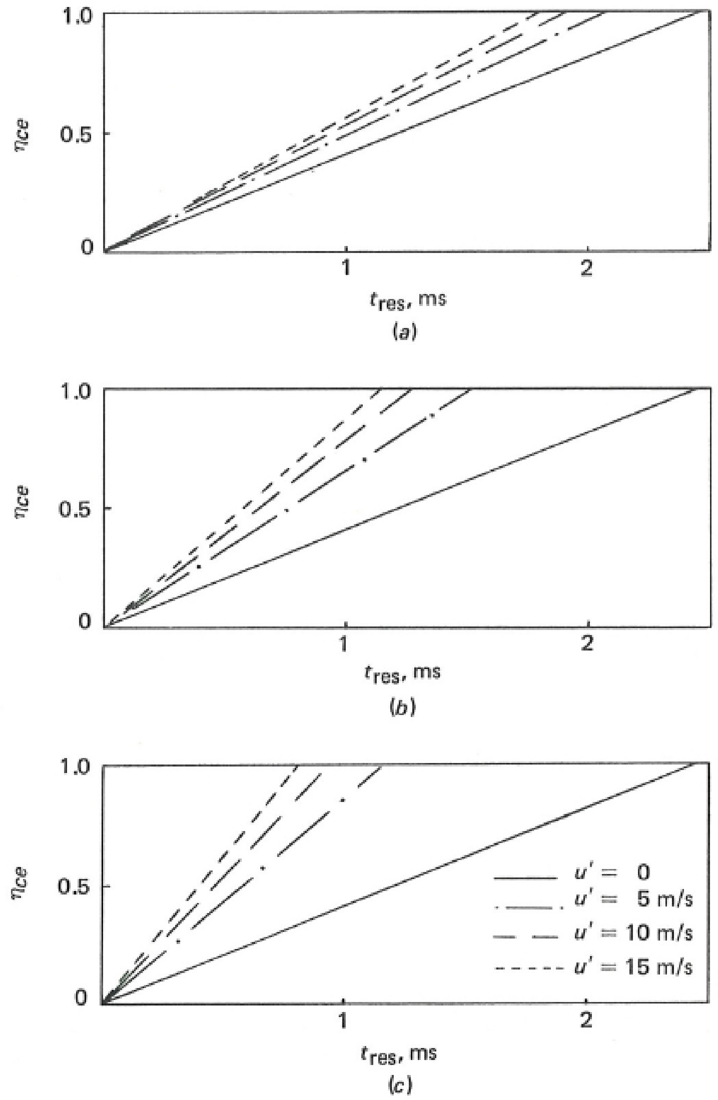



Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 2nd Edition, McGraw Hill

<u>Influence of fuel mean drop size on evaporation rate-controlled combustion efficiency</u>
(Fuel: Diesel Oil)

# Evaporation Rate-Controlled Efficiency:

## Influence of Turbulence and Pressure




Influence of turbulence on evaporation rate-controlled combustion efficiency for three levels of pressure, D = 60μm (a) P=0.1MPa, (b) P=1MPa, (c) P=3MPa)
(Fuel: Kerosene)

Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 2nd Edition, McGraw Hill

# Evaporation Rate-Controlled Efficiency:

## Critical Mean Drop Diameter

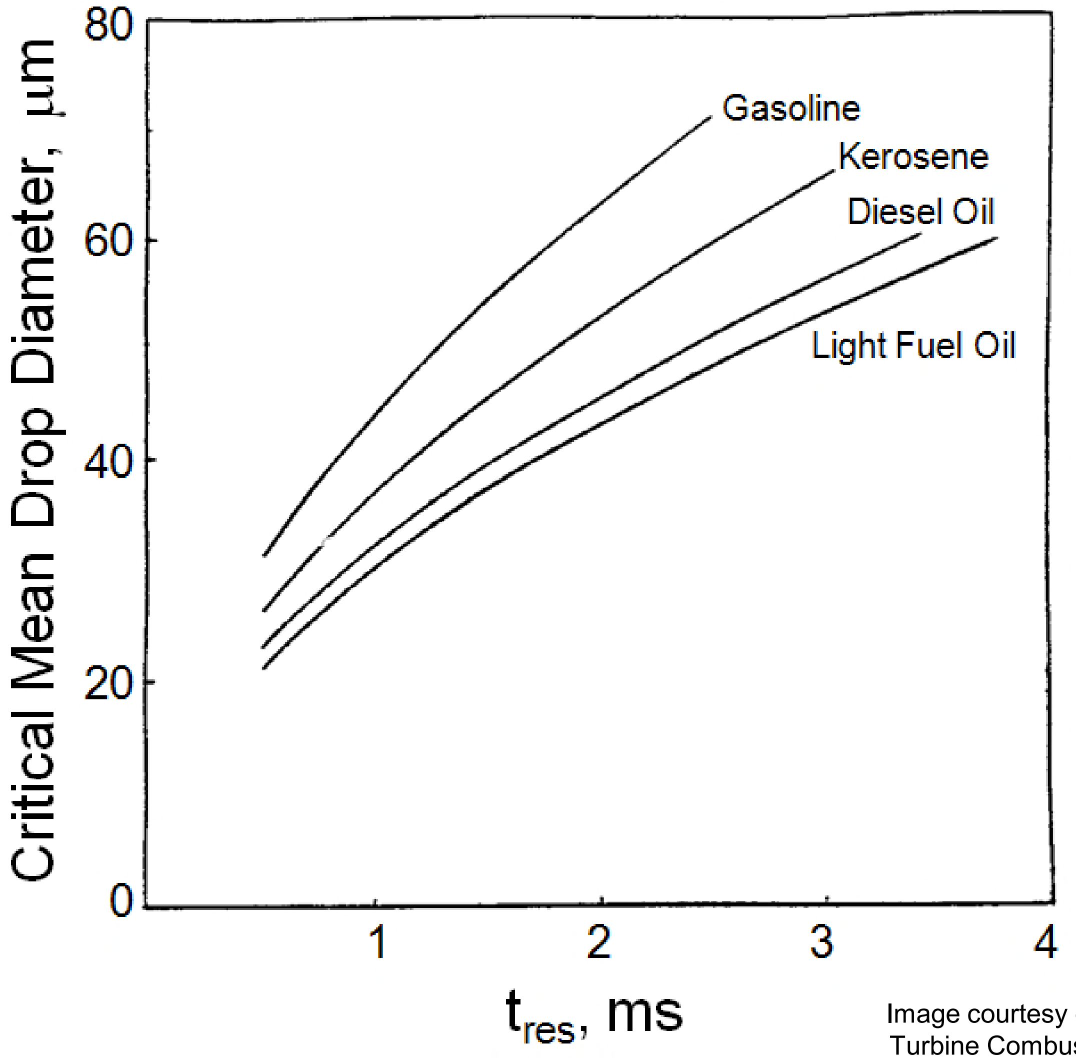
$$\eta_{ce} = \frac{8\,(k/C_p)_g \ln(1+B)\left(1+0.25\,\mathrm{Re}_D^{\,0.5}\right)t_{res}}{\rho_f D^2}$$

- The critical drop diameter is the mean drop size above which evaporation becomes the rate controlling step
  - $\eta_{ce} = 1$
  - For a conservative approach, the term $(1+0.25Re_D^{0.5})$ can be ignored

$$\Rightarrow \qquad D_{crit} = \left[8\left(\frac{k}{Cp}\right)_g \rho_f^{-1} \ln(1+B)\,t_{res}\right]^{0.5}$$

$D_{crit}$ – Critical mean drop diameter

# Evaporation Rate-Controlled Efficiency:
## Critical Mean Drop Diameter




<u>Influence of fuel type and residence time on Critical Drop Diameter</u>

($T_g$ = 2300K)

Image courtesy of: Lefebvre. A. H., 2010, "Gas Turbine Combustion", 2nd Edition, McGraw Hill

# Evaporation Rate-Controlled Efficiency:
## Effect of Fuel Type

$$\eta_{ce} = \frac{8\left(k/C_p\right)_g \ln\left(1+B\right)\left(1+0.25\,\mathrm{Re}_D^{\,0.5}\right)t_{res}}{\rho_f D^2}$$

$$\frac{\eta_{cea}}{\eta_{ceb}} = \frac{\rho_{fb} D_b^{\,2} \ln(1+B_a)}{\rho_{fa} D_a^{\,2} \ln(1+B_b)}$$

a – Corresponds to fuel type a

b – Corresponds to fuel type b

- Assumptions
  - Both fuels burn in the same combustor at the same operating conditions
  - Changes in fluid properties are ignored
  - $\mathrm{Re}_D$ can be ignored as turbulent jet velocities are similar

# Evaporation Rate-Controlled Efficiency:

## Effect of Fuel Type

$$\frac{\eta_{cea}}{\eta_{ceb}} = \frac{\rho_{fb} D_b^{\ 2} \ln(1 + B_a)}{\rho_{fa} D_a^{\ 2} \ln(1 + B_b)}$$

- For swirl atomisers, mean drop size depends on fuel surface tension and viscosity
- Conventional fuels exhibit only slight differences in surface tension

$\Rightarrow$ (from drop size equations) $D \propto \mu_f^{\ 0.25}$

$\mu_f$ – Dynamic viscosity of the fuel

$$\Rightarrow \frac{\eta_{cea}}{\eta_{ceb}} = \frac{\rho_{fb} \mu_{fb}^{\ 0.5} \ln(1 + B_a)}{\rho_{fa} \mu_{fa}^{\ 0.5} \ln(1 + B_b)}$$

# Combustion Efficiency:
## Reaction Rate and Evaporation Rate Controlled Systems

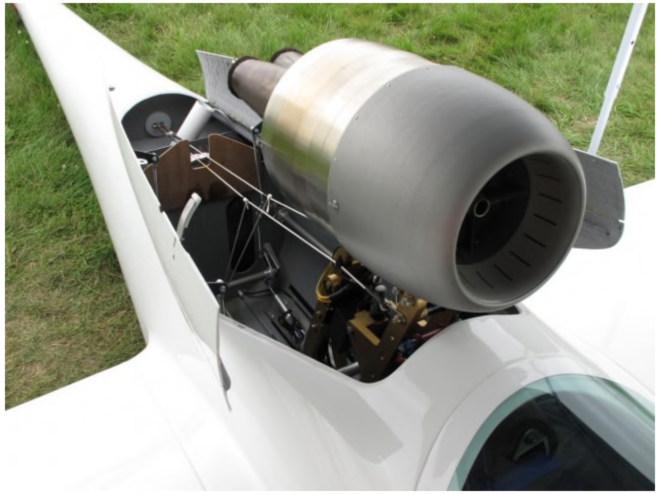
- For some cases (e.g. fuels of low volatility burning at low pressure) the rate of heat release may be limited by both chemical reaction and evaporation rates

$$\Rightarrow \quad \eta_c = \eta_{ce} \eta_{c\theta}$$

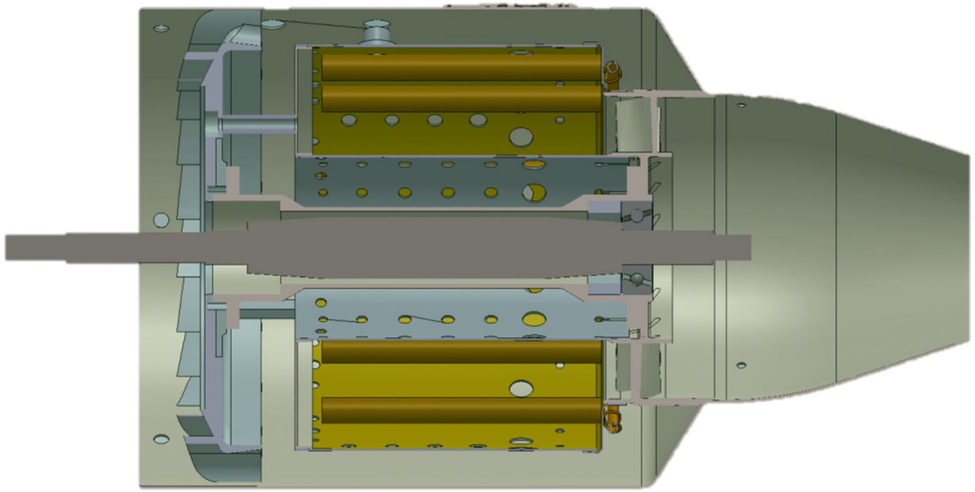
- For $\eta_{ce}$ =1:

$$\eta_c = \eta_{c\theta} = f(\theta) = f\left[ \frac{P_3^{1.75} A_{ref} D_{ref}^{0.75} \exp(T_3 / 300)}{m_a} \right]$$

# Combustion Efficiency:
## Micro Turbojets (CU - CSIR Research Project)



Ref: www.wired.com



Ref: CAT 200KS Project – Internal project Report

**Research program:**

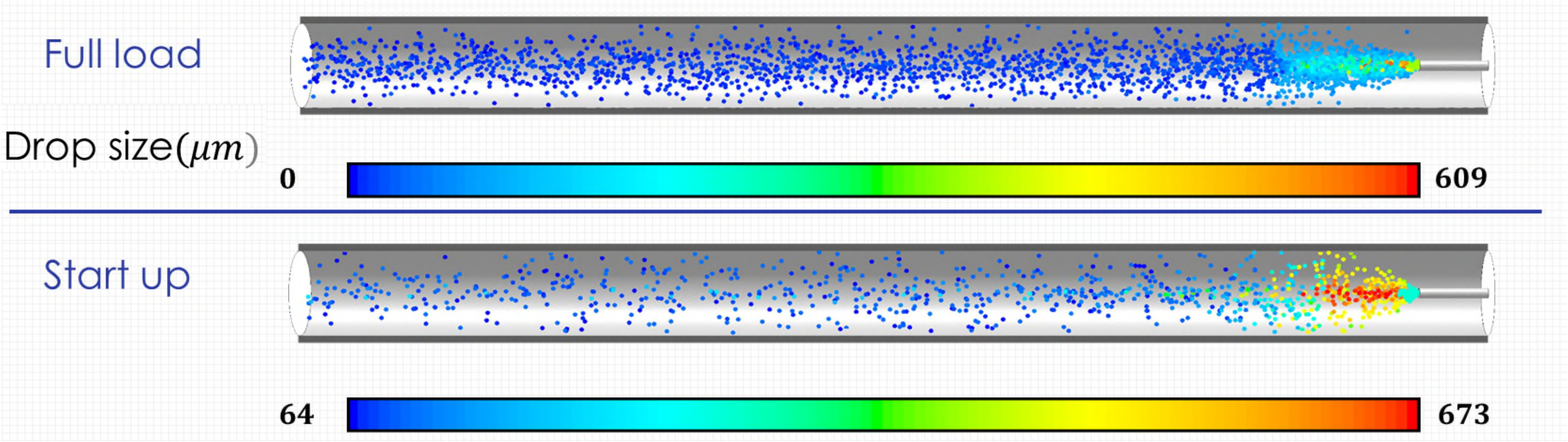
| | |
|---|---|
| Phase 1: | Re-design of compressor and turbine |
| Phase 2: | Re-design of combustion chamber |
| Phase 3: | Test of different injection systems |

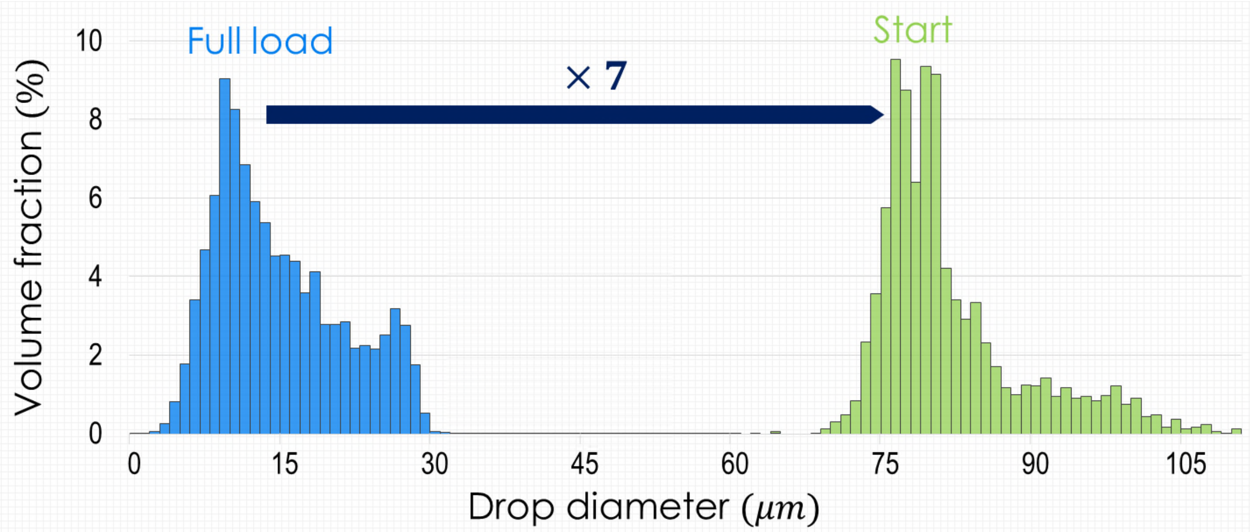
Ignition issues:

Start up failure
Flame exiting the engine

# Combustion Efficiency:

## Micro Turbojets (CU - CSIR Research Project)




Delord, C., “CFD Approach to Investigate Fuel Evaporation in a Micro Turbojet Experiencing Light Up Issues” (2015) MSc Thesis, Propulsion Engineering Centre, Cranfield University

# Combustion Efficiency:

## Tutorial: Numerical Example

An existing gas turbine combustor operates satisfactorily when Aviation Kerosene is used as a fuel. At design point the droplet diameter (SMD) is 70μm and the primary zone mean residence time is 3ms. The gas turbine is to be sold for electric base load power generation and the fuel specified is Light Fuel Oil (LFO). Tests have established that the combustor efficiency is being limited by evaporation when LFO is used but satisfactory with Kerosene.

a. Calculate, first the change in "SMD" required when burning LFO. Assume all other parameters remain unchanged. $D_{LFO} \approx 57mm$ ($\Delta D \approx -13$ mm)

b. Next calculate the change in the "mean residence time" required, again assuming that all other parameters are unchanged. $tres_{LFO} \approx 4.5ms$ ($\Delta tres \approx +1.5ms$)